Algebarski razlomci

(zadaci)

Zadatak 1: Skrati razlomke:

I) $\dfrac{xy}{x-xy}$ **II)** $\dfrac{ab^3}{a^2b-ab^2}$ **III)** $\dfrac{ax-bx}{ax+bx}$

IV) $\dfrac{xz-yz}{z^2+3z}$ **V)** $\dfrac{a^2+a}{ax-ay}$ **VI)** $\dfrac{a^2-2ab}{ab-2b^2}$

VII) $\dfrac{3a^2+4ab}{9a^2b-16b^3}$ **VIII)** $\dfrac{16x^3-36xy^2}{6xy-9y^2}$ **IX)** $\dfrac{2xz-4yz}{5x^3z-20xy^2z}$

X) $\dfrac{12a^5-27a^3b^2}{8a^3b-12a^2b^2}$ **XI)** $\dfrac{2a^4-8a^3b+8a^2b^2}{a^4-2a^3b}$ **XII)** $\dfrac{a^2-6a+9}{a^2-9}$

XIII) $\dfrac{a^2-4}{a^2+a-6}$ **XIV)** $\dfrac{a^2-b^2}{a^3+b^3}$ **XV)** $\dfrac{x^3+y^3+(x+y)^3}{x^2-y^2}$

XVI) $\dfrac{a^3+b^3}{a^4-b^4}$ **XVII)** $\dfrac{a^2-b^2}{a^3+ab^2-a^2b-b^3}$

XVIII) $\dfrac{a^2-b^2}{a^2-a-b-b^2}$ **XIX)** $\dfrac{a^2+2ab+b^2-c^2}{(a+b+c)\,a+(a+b+c)\,c}$

XX) $\dfrac{a^2+b^2-c^2+2ab}{a^2-b^2+c^2+2ac}$ **XXI)** $\dfrac{a^2+6a+5}{a^3+5a^2-a-5}$

XXII) $\dfrac{x^2+2x+2}{(x+1)^4-1}$ **XXIII)** $\dfrac{(2a-3)\,(a-1)^2-4\,(2a-3)}{(a+1)^2\,(a-3)}$

XXIV) $\dfrac{\left(4a^2-4a+1\right)\left(a^2-2a-3\right)}{\left(a^2-6a+9\right)\left[a^2-1+a\,(a+1)\right]}$

XXV) $\dfrac{x^2-4xy+4y^2-4}{x^2-4y^2-2\,(x-2y)}$

XXVI) $\dfrac{\left(a^2-b^2-c^2-2bc\right)\,(a+b-c)}{(a+b+c)\,(a^2-b^2+c^2-2ac)}$

XXVII) $\dfrac{a^2+b^2+c^2+2ab+2bc+2ac}{a^2-b^2-c^2-2bc}$

XXVIII) $\dfrac{x^2-3xy+xz+2y^2-2yz}{a^2-b^2-c^2-2bc}$

XXIX) $\dfrac{a^2-3ab+ac+2b^2-2bc}{a^2-b^2+2bc-c^2}$

XXX) $\dfrac{xy\left(a^2-b^2\right)+abx^2-aby^2}{abx^2+aby^2+xy\left(a^2+b^2\right)}$

XXXI) $\dfrac{3a^2x^2+3b^2-6abx+9a^2x-9ab}{\left(a^3x^3-ab^2x\right)(ax-b+3a)}$

XXXII) $\dfrac{2x^3+3x^2-2x-3y}{2x^2-2x+3y\,(x-1)}$

XXXIII) $\dfrac{3x^3+xy^2-6x^2y-2y^3}{9x^5-xy^4-18x^4y+2y^5}$

XXXIV) $\dfrac{ac+ad+bc+bd-a\,(c+d)}{ac^2-ad^2+bc^2-bd^2}$

XXXV) $\dfrac{2a^4+4a^3b+2a^2b^2+4a^2bc+2a^2c^2+4a^3c}{a^3-ab^2-ac^2-2abc}$

XXXVI) $\dfrac{a^2-a-12}{a^3-13a-12}$ **XXXVI**) $\dfrac{a^2+3a-10}{a^3+4a^2-7a-10}$

XXXIX) $\dfrac{x^3-1}{z^4+z^2+1}$ **XL**) $\dfrac{a^3+b^3+c^3-3abc}{(a-b)^2+(b-c)^2+(c-a)^2}$

Zadatak 2: Ucini naznacene operacije zbrajanja i oduzimanja i dobivene rezultate skrati:

I) $\dfrac{3a-1}{2}+\dfrac{a+2}{3}+\dfrac{1-7a}{4}$ **II**) $\dfrac{a+3}{3}+\dfrac{a+7}{7}+\dfrac{11a}{21}$

III) $\dfrac{x+1}{5}+\dfrac{3+x}{10}+\dfrac{3x-1}{2}$ **IV**) $\dfrac{x-7}{4}+\dfrac{2x+3}{7}-\dfrac{5x-12}{8}$

V) $\dfrac{2y-3}{5}+\dfrac{y-1}{4}-\dfrac{6y-7}{10}$ **VI**) $\dfrac{z-9}{45}-\dfrac{5-z}{15}-\dfrac{z-3}{5}$

VII) $\dfrac{1}{a}+\dfrac{2}{b}+\dfrac{3}{c}$ **VIII**) $\dfrac{x+y}{x^2y}+\dfrac{x-y}{xy^2}$

IX) $\dfrac{a-b}{ab}+\dfrac{b-c}{bc}+\dfrac{c-a}{ca}$ **X**) $\dfrac{1}{a-1}+\dfrac{1}{a+1}$

XI) $\dfrac{1}{a+b}+\dfrac{1}{a-b}-\dfrac{2a}{a^2-b^2}$ **XII**) $\dfrac{1}{x+y}-\dfrac{x}{(x+y)^2}+\dfrac{1-y^2}{(x+y)^3}$

XIII) $\dfrac{3}{x+2}-\dfrac{1}{x-2}+\dfrac{x}{x^2-4}$ **XIV**) $\dfrac{2}{x-3}-\dfrac{3}{x+3}+\dfrac{1+3x}{x^2-9}$

XV) $\dfrac{5x}{x+y}-\dfrac{2x-3y}{x-y}-\dfrac{2\left(x^2-3xy+y^2\right)}{x^2-y^2}$

XVI) $\dfrac{2}{2a-b}+\dfrac{b}{2a+b}-\dfrac{2a^2+3ab}{4a^2-b^2}$ **XVII**) $\dfrac{2}{a-3b}+\dfrac{3}{a+3b}-\dfrac{12b}{a^2-9b^2}$

XVIII) $\frac{1}{a}+\frac{1}{a+1}-\frac{1}{a^2+a}$

XIX) $\frac{x^2}{x^2-xy}+\frac{y^2}{xy-y^2}-\frac{2y}{x-y}$

XX) $\frac{7}{x-1}-\frac{3}{x+1}+\frac{4}{x^2-4x+1}$

XXI) $\frac{a}{a^2+b^2}-\frac{b\,(a-b)^2}{a^4-b^4}$

XXII) $\frac{8x-4x^2}{4-x^2}+\frac{8+4x}{4+4x+x^2}-\frac{2x^2-x^3}{4-x^2}$

XXIII) $\frac{3}{x}+\frac{5}{x+y}-\frac{7x+2y}{x^2+xy}$

XXIV) $\frac{a+1}{3b-4a}+\frac{a-1}{4a+3b}$

XXV) $\frac{x}{x^2-y^2}+\frac{x}{(x-y)^2}$

XXVI) $\frac{a^2+b^2}{ab}-\frac{a^2}{ab+b^2}-\frac{a^2}{a^2+ab}$

XXVII) $1+a+a^2+\frac{a^3}{1-a}$

XXVIII) $\frac{1}{a-2}+\frac{1}{a+3}-\frac{5}{a^2+a-6}$

XXIX) $\frac{2a+1}{2a+3}-\frac{a}{a-2}-\frac{7}{2a^2-a-6}$

XXX) $\frac{1}{a-b}+\frac{b}{a^2+ab+b^2}+\frac{b^2-4ab}{a^3-b^3}$

XXXI) $\frac{1}{2a-1}-\frac{2a}{1+2a+4a^2}-\frac{4a^2}{1-8a^3}$

XXXII) $\frac{a-2b}{a^3-b^3}-\frac{a-b}{a^2b-ab^2+b^3}-\frac{1}{ab+b^2}$

XXXIII) $\frac{2}{y-2}-\frac{y^3-8y^2+8y-16}{y^3-8}$

XXXIV) $\frac{z^2}{4\,(z^3+27)}-\frac{z^2}{4\,(z^3-27)}+\frac{1}{2z^2-18}$

Zadatak 3: Dokazi valjanost ovih jednakosti:

I) $\frac{a-b}{a+b}+\frac{b-c}{b+c}+\frac{c-a}{c+a}+\frac{(a-b)\,(b-c)\,(c-a)}{(a+b)\,(b+c)\,(c+a)}=0$

II) $\frac{1}{(a-b)\,(b-c)}+\frac{1}{(b-c)\,(c-a)}+\frac{1}{(c+a)\,(b-a)}=0$

III) $\frac{a}{(a-2b)\,(a-c)}+\frac{2b}{(2b-c)\,(2b-a)}+\frac{c}{(c-a)\,(c-2b)}=0$

IV) $\frac{a+b}{(b-c)\,(c-a)}+\frac{b+c}{(c-a)\,(a-b)}+\frac{c+a}{(a-b)\,(b-c)}=0$

V) $\dfrac{a^2}{(a-b)(a-c)} + \dfrac{b^2}{(b-c)(b-a)} + \dfrac{c^2}{(c-a)(c-b)} = 1$

VI) $\dfrac{1}{a(a-b)(a-c)} + \dfrac{1}{b(b-c)(b-a)} + \dfrac{1}{c(c-a)(c-b)} = \dfrac{1}{abc}$

Zadatak 4: Reduciraj (pokrati):

I) $\dfrac{42x^3y^2z^2}{35a^2bc} \cdot \dfrac{25abc}{21xyz}$ **II**) $\dfrac{81ac^2}{17b^3d} \cdot \dfrac{34b^2d}{180ac^3}$ **III**) $\dfrac{34a^2b^2c^2}{x^3y^3z} \cdot \dfrac{16x^2y^3z}{17abc}$

IV) $\dfrac{3x^2y}{7ab^2} \cdot 14ab^3$ **V**) $30pq \cdot \dfrac{9ab}{60pqr}$ **VI**) $\dfrac{a^2-1}{a^2-b^2} \cdot \dfrac{a-b}{a+1}$

VII) $\dfrac{x^2+y^2}{x^2-xy} \cdot \dfrac{x^2y-xy^2}{x^4y-y^5}$ **VIII**) $\dfrac{25x^2-1}{9x^2y^2-4y^2} \cdot \dfrac{3xy+2y}{1+5x}$

IX) $\dfrac{a^3-b^3}{a+b} \cdot \dfrac{a^3+b^3}{a-b}$ **X**) $\left(\dfrac{1}{a+1} + \dfrac{1}{a-1}\right) \cdot \dfrac{1}{2a}$

XI) $\left(\dfrac{1}{a+1} + \dfrac{1}{a^2-1}\right) \cdot \dfrac{a+1}{a}$

XII) $\left(\dfrac{1}{2a+3b} + \dfrac{b}{4a^2-9b^2}\right) \cdot \dfrac{2a+3b}{a-b}$

XIII) $(a^2-b^2)\left(\dfrac{a}{b} + \dfrac{b}{a}\right) \cdot \dfrac{b}{a+b}$ **XIV**) $\left[\left(\dfrac{a}{b}-1\right)^2 + \dfrac{a}{b}\right] \cdot \dfrac{a+b}{b}$

XV) $\left(ab + \dfrac{1}{ab}\right) \cdot \left(\dfrac{b-2}{a} + \dfrac{1}{ab}\right)$

XVI) $\left(\dfrac{1}{xy} + \dfrac{1}{x} + \dfrac{1}{y} + 1\right) \cdot \left(\dfrac{1}{xy} - \dfrac{1}{x} - \dfrac{1}{y} + 1\right) + \dfrac{1}{x^2} - \dfrac{1}{x^2y^2} + \dfrac{1-y^2}{y^2}$

XVII) $\dfrac{2(a-3)}{a(2-a)} + \dfrac{a+2}{a+6} \cdot \left(\dfrac{a-2}{a+2} - \dfrac{a+2}{a-2} + \dfrac{9}{a}\right)$

XVIII) $\left[\dfrac{a(16-a)}{a(a^2-4)} - \dfrac{2a+3}{a-2} - \dfrac{2-3a}{a+2}\right] \cdot (a^2+4a+4)$

XIX) $\left(\dfrac{75+12x^2}{25-4x^2} + \dfrac{5+2x}{2x-5} - \dfrac{5-2x}{5+2x}\right) \cdot \dfrac{25-4x^2}{25+4x^2}$

XX) $\left(\dfrac{a}{a-b} - \dfrac{b}{a+b} + \dfrac{2ab}{a^2-b^2}\right) \cdot \left(\dfrac{a}{a-b} - \dfrac{b}{a+b} - \dfrac{2ab}{a^2-b^2}\right)$

XXI) $\left[\left(\dfrac{3a}{2b} - \dfrac{2b}{3a}\right)^2 - \dfrac{8b^2}{9a^2} + 2\right] \cdot \left(\dfrac{1}{9a^2+4b^2} + \dfrac{1}{9a^2-4b^2}\right)$

XXII) $\left[-4a+\dfrac{\left(a^2+4\right)^2-8a^2}{2a}\right]\cdot\left[\dfrac{1}{(a+2)^2}+\dfrac{1}{(a-2)^2}\right]$

XXIII) $\left(\dfrac{x-xy}{x-y}\right)^2+\left(a-\dfrac{z-xy}{x-y}\right)\cdot\left(a+\dfrac{z-xy}{x-y}\right)$

XXIV) $\left[\left(\dfrac{b-a}{a}\right)^2-\dfrac{(a+b)^2-4ab}{a^2-ab}\right]^2\cdot\dfrac{a^4}{a^2b^2-b^4}$

XXV) $\left(\dfrac{2a^2+3a}{4a^2+12a+9}-\dfrac{3a+2}{2a+3}+\dfrac{4a-1}{2a+3}\right)\cdot\dfrac{2a+3}{2a-3}$

XXVI) $\left[\dfrac{a^3-ab^2+b^3}{(a-b)^3}-\dfrac{b}{a-b}\right]\cdot\left(\dfrac{a^2-2ab+b^2}{a^2-ab+b^2}-\dfrac{b}{a}\right)$

XXVII) $\left(\dfrac{a^2-a}{a^2+1}-\dfrac{2a^2}{a^3-a^2+a-1}\right)\cdot\left(1-\dfrac{1}{a^2}\right)$

XXVIII) $\dfrac{2b^2-2b-a+ab}{a^2-4b^2}+\dfrac{b}{2b^2+6b-ab-3a}\cdot\left(b+\dfrac{3b-6}{b-2}\right)$

XXIX) $\left(\dfrac{a^2+b^2}{a^3-a^2b+ab^2-b^3}-\dfrac{2a^2b+2ab^2}{a^4-b^4}\right)\cdot\left(\dfrac{a}{2b^2}+\dfrac{b^2}{2a^3}\right)-\dfrac{b-a}{2a^3}$

XXX) $\left(\dfrac{x^2-xy}{x^2y+y^3}-\dfrac{2x^2}{y^3-xy^2+x^2y-x^3}\right)\cdot\left(1-\dfrac{y-1}{x}-\dfrac{y}{x^2}\right)$

XXXI) $\left(\dfrac{a}{a-2b}+\dfrac{b}{a+2b}\right)\cdot\dfrac{a^3+8b^3}{a^3+3a^2b-2ab^2}$

XXXII) $\left(\dfrac{1}{a^2+3a+1}+\dfrac{2a}{a^2+4a+3}+\dfrac{1}{a^2+5a+6}\right)\cdot\dfrac{(a-3)^2+12a}{2}$

XXXIII) $4x^2+2x+1+\left(8x^2-1\right)\cdot\left(\dfrac{2x+4x^2}{4x^2+1}-\dfrac{2x+1}{2x-1}\right)$

XXXIV) $\left(\dfrac{a}{a^2+2a+4}+\dfrac{a^2+8}{a^3-8}-\dfrac{1}{a-2}\right)\cdot\left(\dfrac{a^2}{a^2-4}-\dfrac{2}{2-a}\right)$

XXXV) $\left[\dfrac{(x+y)^2+2y^2}{x^3-y^3}-\dfrac{1}{x-y}+\dfrac{x+y}{x^2+xy+y^2}\right]\cdot\left(\dfrac{1}{y}-\dfrac{1}{x}\right)$

Zadatak 5: Pojednostavni:

I) $\dfrac{28a^2b}{5x^2y^3}:\dfrac{35a^2b^2}{30xy^2}$ **II)** $\dfrac{15x^2y^5}{20a^2b^3}:\dfrac{5x^2y^4}{40ab^2}$ **III)** $\dfrac{9x^3y}{15a^3b}:\dfrac{36xy}{75ab}$

IV) $\dfrac{27abc}{60xyz}:\dfrac{9a^2b^2c^2}{15x^2y}$ **V)** $\dfrac{a^2-4}{9a^2-16}:\dfrac{a+2}{3a-4}$

VI) $\dfrac{2xy+2y^2}{12x-16y} : \dfrac{4x^2-4y^2}{16x^2-4y^2}$ **VII)** $\dfrac{\left(a^2+ab\right)^2}{a^2-b^2} : \dfrac{(a+b)^2}{\left(ab-b^2\right)^2}$

VIII) $\dfrac{(a+b)^2-c^2}{(a-b)^2-c^2} : \dfrac{a+b+c}{a-b+c}$ **IX)** $\dfrac{(a+b)^2-c^2}{(a-b)^2-c^2} : \dfrac{(a+c)^2-b^2}{(a-c)^2-b^2}$

X) $\dfrac{3ax-bx-2by+6ay}{2x+y} : \dfrac{6ax-2bx-by+3ay}{x+2y}$

XI) $\dfrac{xy+(x-y)^2}{(x+y)^2-xy} : \dfrac{x^5+y^5+x^2y^3+x^3y^2}{\left(x^3+y^3+x^2y+xy^2\right)\left(x^3-y^3\right)}$

XII) $\dfrac{x^4-y^4}{x^2+2xy+y^2} : \dfrac{x^3+xy^2}{x+y}$

XIII) $\left[\dfrac{a^2-b^2}{ab} - \dfrac{1}{a+b}\left(\dfrac{a^2}{b}-\dfrac{b^2}{a}\right)\right] : \dfrac{a-b}{b}$

XIV) $\left(\dfrac{3}{2a-b} - \dfrac{2}{2a+b} - \dfrac{1}{2a-b}\right) : \dfrac{b^2}{4a^2-b^2}$

XV) $\left[\dfrac{1}{a^2+2ab+b^2} + \dfrac{1}{a^2-b^2} - \dfrac{1}{(b-a)^2}\right] : \dfrac{b^2+4ab-a^2}{a^2-b^2}$

XVI) $\left[\dfrac{x-1}{3x+(x-1)^2} - \dfrac{1-3x+x^2}{x^3-1} - \dfrac{1}{x-1}\right] : \dfrac{x^2+1}{1-x}$

XVII) $\left[\dfrac{a^2}{a^2-b^2} - \dfrac{a^2b}{a^3+b^2}\cdot\left(\dfrac{a}{ab+b^2}+\dfrac{b}{a^2+ab}\right)\right] : \dfrac{b}{a-b}$

XVIII) $\left[\dfrac{2}{(a+b)^3}\cdot\left(\dfrac{1}{a}+\dfrac{1}{b}\right) + \dfrac{1}{a^2+2ab+b^2}\cdot\left(\dfrac{1}{a^2}+\dfrac{1}{b^2}\right)\right] : \dfrac{a-b}{a^3-b^3}$

XIX) $\left[\dfrac{y^2+z^2}{y^2z^2}\cdot\left(\dfrac{1}{y^2}-\dfrac{1}{z^2}\right) - \left(\dfrac{1}{x^2}-\dfrac{1}{z^2}\right)\cdot\dfrac{x^2+z^2}{x^2z^2}\right] : \dfrac{x^2+y^2}{x^2y^2}$

XX) $\left(\dfrac{3x+2}{3x^2+1} - \dfrac{18x^3-x-9}{9x^4-1} + \dfrac{3x-2}{3x^2-1}\right) : \dfrac{x^2+10x+25}{9x^4-1}$

XXI) $\left[\dfrac{2}{(a+y)^3}\left(\dfrac{1}{x}+\dfrac{1}{y}\right) + \dfrac{1}{x^2+2xy+y^2}\left(\dfrac{1}{x^2}+\dfrac{1}{y^2}\right)\right] : \dfrac{x-y}{x^3y^3}$

XXII) $\left[\left(\dfrac{x-y}{4y}+\dfrac{5y}{4x+4y}\right)\cdot\dfrac{x}{x+2y} - \dfrac{x^2-xy+6y^2}{4xy+8y^2}\right] : \dfrac{x+6y}{4x+8y}$

XXIII) $\left(3x+\dfrac{3}{x+1}-2\right) : \left(\dfrac{x+4}{x+1}-1+5x+6x^2\right)$

XXIV) $\left(\dfrac{a-b}{a^2+ab} - \dfrac{a}{ab+b^2}\right) : \left(\dfrac{b^2}{a^3-ab^2}+\dfrac{1}{a+b}\right)$

XXV) $\left(a-\dfrac{4ab}{a+b}+b\right):\left(\dfrac{a}{a+b}-\dfrac{b}{b-a}-\dfrac{2ab}{a^2-b^2}\right)$

XXVI) $\dfrac{a^4-b^4}{a^2b^2}:\left[\left(1+\dfrac{a^2}{b^2}\right)\left(1-\dfrac{2a}{b}-\dfrac{a^2}{b^2}\right)\right]$

XXVII) $\left(\dfrac{5a^2-6a+3}{a-1}+4a^2-a+2\right):\left(2a+1+\dfrac{2a}{a-1}\right)$

XXVIII) $\left(\dfrac{1+x}{1-x}-\dfrac{1-x}{1+x}+\dfrac{4x^2}{x^2-1}\right):\left(\dfrac{2-2x}{x^2}+2-\dfrac{2}{x^3+x^2}\right)$

XXIX) $\left(1-\dfrac{2}{1-3x}\right)\left(1-\dfrac{9x-9x^2}{3x+1}\right):2\left(1-9x^2\right)$

XXX) $\left[\left(\dfrac{2}{a}-\dfrac{a}{4}-\dfrac{3}{2a}+\dfrac{1}{2a}\right):\dfrac{2-a}{4}-\left(1+\dfrac{2}{a}\right)\right]:\dfrac{a}{4-a}$

XXXI) $\left(\dfrac{x}{x+y}-\dfrac{x^2}{x^2-y^2}\right):\left(\dfrac{x^2}{x+y}-\dfrac{x^3}{x^2+2xy+y^2}\right)$

XXXII) $\left(2a-\dfrac{4a-1}{2a}\right)\cdot\dfrac{4a^2}{1-4a^2}:\left(\dfrac{1}{a}-\dfrac{4}{1+2a}\right)$

XXXIII) $\left(\dfrac{a}{a+2}+\dfrac{1}{a^2-4}\right)\cdot\left(\dfrac{a+1}{a-1}+\dfrac{2a+5}{1-a^2}\right):\left(\dfrac{1}{a+1}-\dfrac{1}{2a}\right)$

XXXIV) $\left(\dfrac{x}{x+1}-\dfrac{x^2}{x^2+2a+1}\right):\left(\dfrac{x}{x^2-1}-\dfrac{1}{x+1}\right)\cdot\dfrac{x+1}{x}$

XXXV) $6a+\left(\dfrac{a}{a-2}-\dfrac{a}{a+2}\right):\dfrac{4a}{a^4-2a^3+8a-16}$

XXXVI) $\left(\dfrac{x+y}{y}-\dfrac{2y}{y-x}\right)\cdot\dfrac{y-x}{x^2+y^2}+\left(\dfrac{x^2+1}{2x-1}-\dfrac{x}{2}\right):\dfrac{x+2}{1-2x}$

XXXVII) $\left(\dfrac{x^2+y^2}{xy}-2\right):\left(\dfrac{2x^2+2xy}{x^2+2xy+y^2}-1\right)\cdot\left(\dfrac{1}{x+y}+\dfrac{1}{x-y}\right)$

XXXVIII) $\left[\left(\dfrac{a}{b}-\dfrac{b}{a}\right):(a+b)+a\left(\dfrac{1}{b}+\dfrac{1}{a}\right)\right]:\dfrac{1+a}{b}$

XXXIX) $\left(\dfrac{x+2}{x-2}\right)^2:\dfrac{x^3+4x^2+4x}{3x^2-12x+12}\cdot\dfrac{x}{3}$

XL) $\left(\dfrac{1}{1-a}+\dfrac{1}{\dfrac{1}{a}-1}\right):\dfrac{\dfrac{a+1}{2}}{\dfrac{a+1}{2}-1}$

XLI) $\left(\dfrac{a^2}{b^2}+\dfrac{b}{a}\right):\left(\dfrac{a}{b^2}-\dfrac{1}{b}+\dfrac{1}{a}\right):\dfrac{(a-b)^2+4ab}{1+\dfrac{a}{b}}$

XLII) $1+\dfrac{x^3+1}{x^3-x^2}:\left[\dfrac{1}{x^2}+\dfrac{1}{x}\cdot\dfrac{x+1}{(x-1)^2}-\dfrac{1}{(1-x)^2}\right]$

XLIII) $\dfrac{2a-\dfrac{1}{2a+1}+1}{1+\dfrac{1}{2a+1}}$ **XLIV**) $\dfrac{x^2+\dfrac{1}{x}}{x+\dfrac{1}{x}-1}$ **XLV**) $\dfrac{\dfrac{x}{a}-\dfrac{x}{2a}}{\dfrac{x^2}{a^2}-\dfrac{x^2}{2a^2}}$

XLVI) $\dfrac{x-\dfrac{x-1}{x+1}}{1+\dfrac{x(x-1)}{x+1}}$ **XLVII**) $\dfrac{\left(\dfrac{1}{x}+\dfrac{1}{y}-\dfrac{2z}{xy}\right)(a+y+2z)}{\dfrac{1}{x^2}+\dfrac{1}{y^2}+\dfrac{2}{xy}-\dfrac{4z^2}{x^2y^2}}$

XLVIII) $\dfrac{\left(\dfrac{x}{y}+\dfrac{y}{x}+1\right)\left(\dfrac{1}{x}-\dfrac{1}{y}\right)}{\dfrac{x^2}{y^2}+\dfrac{y^2}{x^2}-\left(\dfrac{x}{y}-\dfrac{y}{x}\right)}$ **XLIX**) $\dfrac{\left(\dfrac{x}{y}+\dfrac{y}{x}-1\right)\left(\dfrac{x}{y}+\dfrac{y}{x}+1\right)}{\left(\dfrac{x^4}{y^2}+\dfrac{y^4}{x^2}\right):(x^2-y^2)}$

L) $\dfrac{(2x-y)^2}{4x^2+2xy-\dfrac{(2x+y)^2}{\dfrac{2x}{y}+1}}$ **LI**) $\dfrac{\left(\dfrac{1}{a}+\dfrac{1}{b}-\dfrac{x}{ab}\right)(x+a+b)}{\dfrac{1}{a^2}+\dfrac{1}{b^2}+\dfrac{2}{ab}-\dfrac{x^2}{a^2b^2}}$

LII) $\dfrac{\dfrac{1}{1-a}+\dfrac{1}{1+a}}{\dfrac{1}{1-a}-\dfrac{1}{1+a}}$ **LIII**) $\dfrac{\dfrac{a-b}{a+b}+\dfrac{a+b}{a-b}}{\dfrac{a}{b}+\dfrac{b}{a}}$ **LIV**) $\dfrac{\dfrac{a-b}{a+b}+\dfrac{a+b}{a-b}}{\dfrac{a+b}{a-b}-\dfrac{a-b}{a+b}}$

LV) $\dfrac{\dfrac{2a+b}{2a-b}-\dfrac{2a-b}{2a+b}}{\dfrac{4a^2+b^2}{4a^2-b^2}-\dfrac{4a^2-b^2}{4a^2+b^2}}$ **LVI**) $\dfrac{\dfrac{a+b}{a-b}-\dfrac{a-b}{a+b}}{1-\dfrac{a-b}{a+b}}$

LVII) $\dfrac{\dfrac{a}{1+\dfrac{1}{a}}+1-\dfrac{1}{1+a}}{\dfrac{a}{1-\dfrac{1}{a}}-a+\dfrac{1}{1-a}}$ **LVIII**) $\dfrac{\dfrac{1}{a}-\dfrac{1}{b+c}}{\dfrac{1}{a}+\dfrac{1}{b+c}}\cdot\dfrac{\dfrac{1}{b}+\dfrac{1}{a+c}}{\dfrac{1}{b}-\dfrac{1}{a+c}}$

LIX) $\dfrac{\dfrac{1-x}{1-x+x^2}+\dfrac{1+x}{1+x+x^2}}{\dfrac{1+x}{1+x+x^2}-\dfrac{1-x}{1-x+x^2}}$ **LX**) $\dfrac{x}{x+\dfrac{1}{x+\dfrac{1}{x}}}\cdot\dfrac{x+\dfrac{2}{x}}{x+\dfrac{1}{x}}$

LXI) $$\frac{\dfrac{x-a}{1+xa}-\dfrac{x-b}{1+xb}}{1+\dfrac{(x-a)(x-b)}{(1+xa)(1+xb)}}$$

LXII) $$\frac{\dfrac{x+y}{x-y}-\dfrac{x-y}{x+y}}{\dfrac{x+y}{x-y}+\dfrac{x-y}{x+y}}:\frac{x^2y^2}{(x+y)^2+(x-y)^2}$$

LXIII) $$1+\frac{x}{1+\dfrac{x}{1+\dfrac{x}{1-x}}}$$

LXIV) $$\frac{1+\dfrac{1+\dfrac{1+x}{1-3x}}{1-3\dfrac{1+x}{1-3x}}}{1-3\dfrac{1+\dfrac{1+x}{1-3x}}{1-3\dfrac{1+x}{1-3x}}}$$

LXV) $$\left\{\frac{\left[\dfrac{(p-1)(px-qx)}{x-px}+r\right]\cdot(p-x)}{p(q+r-p)}+\frac{p-x}{p}\right\}:\frac{2\left(p^2-x^2\right)}{p^2+px}$$

LXVI) $$\left\{\left(1-\frac{1-\dfrac{y}{x+y}}{1-\dfrac{x}{x+y}}\right):\left[y^2-\frac{x}{1+\dfrac{x}{y-x}}\left(\frac{xy}{y-x}-x\right)\right]\right\}\cdot\frac{x^2+xy+y^2}{y-x}$$

LXVII) $$\left\{\frac{\left[\dfrac{(a-1)(ab-bc)}{b-ab}+d\right]\cdot(a-b)}{a(c+d-a)}+\frac{a-b}{a}\right\}\cdot\frac{a^2+ab}{2(a^2-b^2)}$$

LXVIII) $$\left\{\left[1+\frac{1+\dfrac{a^2-b^2}{a^2+b^2}}{1-\dfrac{a^2-b^2}{a^2+b^2}}\right]\cdot\frac{1}{1+\dfrac{a^2}{b^2}}+\frac{a^2-b^2}{a-b}\right\}\cdot\frac{a}{1+a+b}$$

<u>Zadatak 6</u>: Dokazi ove jednakosti:

I) $$\frac{b^2c^2}{(a-b)(a-c)}+\frac{c^2a^2}{(b-c)(b-a)}+\frac{a^2b^2}{(c-a)(c-b)}=ab+bc+ca$$

II) $$\frac{a^2-bc}{(a+b)(a+c)}+\frac{b^2-ac}{(b+c)(a+b)}+\frac{c^2-ab}{(a+c)(b+c)}=0$$

III) $\dfrac{a^3}{(a-b)(a-c)}+\dfrac{b^3}{(b-c)(b-a)}+\dfrac{c^3}{(c-a)(c-b)}=a+b+c$

IV) $\left(1+\dfrac{c}{a+b}\right)\left(1+\dfrac{a}{b+c}\right)\left(1+\dfrac{b}{a+c}\right)-\dfrac{a^3+b^3+c^3}{(a+b)(b+c)(c+a)}=3$

V) $\dfrac{a}{(a-b)(a-c)(a-d)}+\dfrac{b}{(b-a)(b-c)(b-d)}+$

$$+\frac{c}{(c-a)(c-b)(c-d)}+\frac{d}{(d-a)(d-b)(d-c)}=0$$

VI) $\dfrac{a^2}{(a-b)(a-c)(a-d)}+\dfrac{b^2}{(b-a)(b-c)(b-d)}+$

$$+\frac{c^2}{(c-a)(c-b)(c-d)}+\frac{d^2}{(d-a)(d-b)(d-c)}=0$$

VII) $\dfrac{a^3}{(a-b)(a-c)(a-d)}+\dfrac{b^3}{(b-a)(b-c)(b-d)}+$

$$+\frac{c^3}{(c-a)(c-b)(c-d)}+\frac{d^3}{(d-a)(d-b)(d-c)}=1$$

VIII) $\dfrac{1}{1-a}+\dfrac{1}{1+a}+\dfrac{2}{1+a^2}+\dfrac{4}{1+a^4}+\dfrac{8}{1+a^8}+\dfrac{16}{1+a^{16}}=\dfrac{32}{1-a^{32}}$

IX) $\dfrac{1}{a(a+1)}+\dfrac{1}{(a+1)(a+2)}+\dfrac{1}{(a+2)(a+3)}+\dfrac{1}{(a+3)(a+4)}+$

$$+\frac{1}{(a+4)(a+5)}=\frac{5}{a(a+5)}$$

X) $\dfrac{\dfrac{a^2(c-b)}{bc}+\dfrac{b^2(a-c)}{ac}+\dfrac{x^2(b-a)}{ab}}{\dfrac{a(c-b)}{bc}+\dfrac{b(a-c)}{ac}+\dfrac{c(b-a)}{ab}}=a+b+c$

XI) $\dfrac{b-c}{(a-b)(a-c)}+\dfrac{c-a}{(b-c)(b-a)}+\dfrac{a-b}{(c-a)(c-b)}=\dfrac{2}{a-b}+\dfrac{2}{b-c}+\dfrac{2}{c-a}$